Pre Calculus Honors
Unit 6 Trigonometric Properties and Equations

1. To prepare for your test **add content to your previous personal study guide.** Use the index below to pull out important topics from Unit 6

6.1

A. Fundamental Properties - Reciprocal and Quotient
B. Fundamental Properties - Pythagorean
C. Fundamental Properties - Even/Odd

6.2

A. Sum and Difference Properties for Sine and Cosine
B. Sum and Difference Properties for Tangent
C. Application problems for Sum and Difference Properties

6.3

A. Double Angle Formulas
B. Power Reducing Formulas
C. Half Angle Formulas

6.4

A. Basic Trig equations
B. Trig equations with more than one function
C. Trig equations that might require properties

2 . **Complete at least fifteen problems** from the bank on the back of this study guide. Questions 43 - 49 cover material you are **not** responsible for. Focus on the areas you need to practice, but make sure you look over all types of problems/topics. *The answer key is attached. I will be checking your work for the fifteen problems.* This guide is meant to be a general, *basic* review of the topics covered in Unit 6

3. **Review** your quizzes, homework, classwork practice, and warm-ups. Practice specifically with the problems you missed!

Your study guide (3 points) and the practice problems (2 points) are homework grades.

Your test is on Tuesday, May 24 and Wednesday, May 25. You are welcome to attend a tutorial or stop by during lunch (any day but Tuesday) with specific questions or areas to review. <u>Please let Mrs. Pike know in advance that you are coming!</u>

DEFINITIONS AND CONCEPTS	EXAMPL
5.5 Trigonometric Equations	
a. The values that satisfy a trigonometric equation are its solutions.	Ex. 1, p. 6
b. To solve an equation containing a single trigonometric function, isolate the function on one side and solve for the variable.	Ex. 2, p. Ex. 3, p.
c. When solving equations involving multiple angles, the period plays an important role in ensuring that we do not leave out any solutions.	Ex. 4, p. Ex. 5, p.
d. Trigonometric equations quadratic in form can be expressed as $au^2 + bu + c = 0$, where u is a trigonometric function and $a \neq 0$. Such equations can be solved by factoring, the square root property, or the quadratic formula.	Ex. 12, p.
e. Factoring can be used to separate two different trigonometric functions in an equation.	Ex. 6, p.
f. Identities are used to solve some trigonometric equations.	Ex. 7, p. Ex. 8, p. Ex. 9, p. Ex. 10, p.
g. Some trigonometric equations have solutions that cannot be determined by knowing the exact values of trigonometric functions of special angles. Such equations are solved using a calculator's inverse trigonometric function feature.	Ex. 11, Ex. 12,

Review Exercises

5.1

In Exercises 1–13, verify each identity.

1. $\sec x - \cos x = \tan x \sin x$
2. $\cos x + \sin x \tan x = \sec x$
3. $\sin^2\theta(1 + \cot^2\theta) = 1$
4. $(\sec\theta - 1)(\sec\theta + 1) = \tan^2\theta$
5. $\dfrac{1 - \tan x}{\sin x} = \csc x - \sec x$
6. $\dfrac{1}{\sin t - 1} + \dfrac{1}{\sin t + 1} = -2\tan t \sec t$
7. $\dfrac{1 + \sin t}{\cos^2 t} = \tan^2 t + 1 + \tan t \sec t$
8. $\dfrac{\cos x}{1 - \sin x} = \dfrac{1 + \sin x}{\cos x}$
9. $1 - \dfrac{\sin^2 x}{1 + \cos x} = \cos x$
10. $(\tan\theta + \cot\theta)^2 = \sec^2\theta + \csc^2\theta$
11. $\dfrac{1}{\sin\theta + \cos\theta} + \dfrac{1}{\sin\theta - \cos\theta} = \dfrac{2\sin\theta}{\sin^4\theta - \cos^4\theta}$
12. $\dfrac{\cos t}{\cot t - 5\cos t} = \dfrac{1}{\csc t - 5}$
13. $\dfrac{1 - \cos t}{1 + \cos t} = (\csc t - \cot t)^2$

5.2 and 5.3

In Exercises 14–19, use a sum or difference formula to find the exact value of each expression.

14. $\cos(45° + 30°)$
15. $\sin 195°$
16. $\tan\left(\dfrac{4\pi}{3} - \dfrac{\pi}{4}\right)$
17. $\tan\dfrac{5\pi}{12}$
18. $\cos 65°\cos 5° + \sin 65°\sin 5°$
19. $\sin 80°\cos 50° - \cos 80°\sin 50°$

In Exercises 20–31, verify each identity.

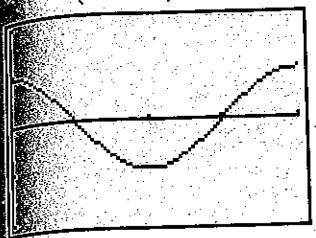
20. $\sin\left(x + \dfrac{\pi}{6}\right) - \cos\left(x + \dfrac{\pi}{3}\right) = \sqrt{3}\sin x$
21. $\tan\left(x + \dfrac{3\pi}{4}\right) = \dfrac{\tan x - 1}{1 + \tan x}$
22. $\sec(\alpha + \beta) = \dfrac{\sec\alpha\sec\beta}{1 - \tan\alpha\tan\beta}$
23. $\dfrac{\cos(\alpha - \beta)}{\cos\alpha\cos\beta} = 1 + \tan\alpha\tan\beta$
24. $\cos^4 t - \sin^4 t = \cos 2t$
25. $\sin t - \cos 2t = (2\sin t - 1)(\sin t + 1)$
26. $\dfrac{\sin 2\theta - \sin\theta}{\cos 2\theta + \cos\theta} = \dfrac{1 - \cos\theta}{\sin\theta}$
27. $\dfrac{\sin 2\theta}{1 - \sin^2\theta} = 2\tan\theta$
28. $\tan 2t = 2\sin t\cos t\sec 2t$
29. $\cos 4t = 1 - 8\sin^2 t\cos^2 t$
30. $\tan\dfrac{x}{2}(1 + \cos x) = \sin x$
31. $\tan\dfrac{x}{2} = \dfrac{\sec x - 1}{\tan x}$

In Exercises 32–34, the graph with the gi... a $\left[0, 2\pi, \dfrac{\pi}{2}\right]$ by $[-2, 2, 1]$ viewing rectangl...

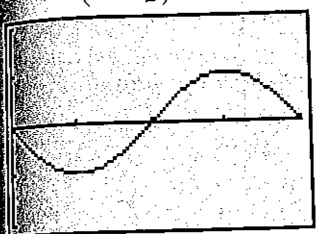
a. *Describe the graph using anothe...*

b. *Verify that the two equations are...*

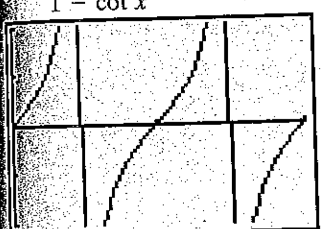
32. $y = \sin\left(x - \frac{3\pi}{2}\right)$



33. $y = \cos\left(x + \frac{\pi}{2}\right)$



34. $y = \dfrac{\tan x - 1}{1 - \cot x}$



In Exercises 35–38, find the exact value of the following under the given conditions:

a. $\sin(\alpha + \beta)$

b. $\cos(\alpha - \beta)$

c. $\tan(\alpha + \beta)$

d. $\sin 2\alpha$

e. $\cos\dfrac{\beta}{2}$

35. $\sin \alpha = \frac{3}{5}, 0 < \alpha < \frac{\pi}{2}$, and $\sin \beta = \frac{12}{13}, \frac{\pi}{2} < \beta < \pi$.

36. $\tan \alpha = \frac{4}{3}, \pi < \alpha < \frac{3\pi}{2}$, and $\tan \beta = \frac{5}{12}, 0 < \beta < \frac{\pi}{2}$.

37. $\tan \alpha = -3, \frac{\pi}{2} < \alpha < \pi$, and $\cot \beta = -3, \frac{3\pi}{2} < \beta < 2\pi$.

38. $\sin \alpha = -\frac{1}{3}, \pi < \alpha < \frac{3\pi}{2}$, and $\cos \beta = -\frac{1}{3}, \pi < \beta < \frac{3\pi}{2}$.

In Exercises 39–42, use double- and half-angle formulas to find the exact value of each expression.

39. $\cos^2 15° - \sin^2 15°$

40. $\dfrac{2\tan\frac{5\pi}{12}}{1 - \tan^2\frac{5\pi}{12}}$

41. $\sin 22.5°$

42. $\tan\dfrac{\pi}{12}$

5.4

In Exercises 43–44, express each product as a sum or difference.

43. $\sin 6x \sin 4x$

44. $\sin 7x \cos 3x$

In Exercises 45–46, express each sum or difference as a product. If possible, find this product's exact value.

45. $\sin 2x - \sin 4x$

46. $\cos 75° + \cos 15°$

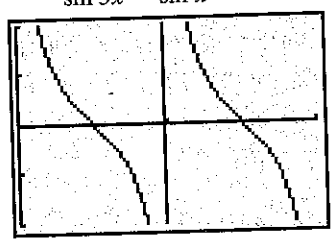
In Exercises 47–48, verify each identity.

47. $\dfrac{\cos 3x + \cos 5x}{\cos 3x - \cos 5x} = \cot x \cot 4x$

48. $\dfrac{\sin 2x + \sin 6x}{\sin 2x - \sin 6x} = -\tan 4x \cot 2x$

49. The graph with the given equation is shown in a $\left[0, 2\pi, \frac{\pi}{2}\right]$ by $[-2, 2, 1]$ viewing rectangle.

$$y = \frac{\cos 3x + \cos x}{\sin 3x - \sin x}$$



a. Describe the graph using another equation.

b. Verify that the two equations are equivalent.

5.5

In Exercises 50–53, find all solutions of each equation.

50. $\cos x = -\dfrac{1}{2}$

51. $\sin x = \dfrac{\sqrt{2}}{2}$

52. $2 \sin x + 1 = 0$

53. $\sqrt{3} \tan x - 1 = 0$

In Exercises 54–67, solve each equation on the interval $[0, 2\pi)$. Use exact values where possible or give approximate solutions correct to four decimal places.

54. $\cos 2x = -1$

55. $\sin 3x = 1$

56. $\tan\dfrac{x}{2} = -1$

57. $\tan x = 2 \cos x \tan x$

58. $\cos^2 x - 2\cos x = 3$

59. $2\cos^2 x - \sin x = 1$

60. $4\sin^2 x = 1$

61. $\cos 2x - \sin x = 1$

62. $\sin 2x = \sqrt{3} \sin x$

63. $\sin x = \tan x$

64. $\sin x = -0.6031$

65. $5\cos^2 x - 3 = 0$

66. $\sec^2 x = 4 \tan x - 2$

67. $2\sin^2 x + \sin x - 2 = 0$

68. A ball on a spring is pulled 6 inches below its rest position and then released. After t seconds, the ball's distance, d, in inches from its rest position is given by

$$d = -6 \cos\frac{\pi}{2}t.$$

Find all values of t for which the ball is 3 inches below its rest position.

69. You are playing catch with a friend located 100 feet away. If you throw the ball with an initial velocity of $v_0 = 90$ feet per second, at what angle of elevation, θ, to the nearest degree should you direct your throw so that it can be caught easily? Use the formula

$$d = \frac{v_0^2}{16} \sin\theta \cos\theta.$$

Chapter 5 Review Exercises

For Exercises 1–13, proofs may vary.

14. $\frac{\sqrt{6}-\sqrt{2}}{4}$ **15.** $\frac{\sqrt{2}-\sqrt{6}}{4}$ **16.** $2-\sqrt{3}$ **17.** $\sqrt{3}+2$ **18.** $\frac{1}{2}$ **19.** $\frac{1}{2}$

For Exercises 19–31, proofs may vary.

32. a. $y = \cos x$ **b.** $\sin\left(x - \frac{3\pi}{2}\right) = \sin x \cos\frac{3\pi}{2} - \cos x \sin\frac{3\pi}{2} = \sin x \cdot 0 - \cos x \cdot -1 = \cos x$

33. a. $y = -\sin x$ **b.** $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2} = \cos x \cdot 0 - \sin x \cdot 1 = -\sin x$

34. a. $y = \tan x$ **b.** $y = \frac{\tan x - 1}{1 - \cot x} = \frac{\frac{\sin x}{\cos x} - 1}{1 - \frac{\cos x}{\sin x}} = \frac{\frac{\sin x - \cos x}{\cos x}}{\frac{\sin x - \cos x}{\sin x}} = \frac{\sin x - \cos x}{\cos x} \cdot \frac{\sin x}{\sin x - \cos x} = \frac{\sin x}{\cos x} = \tan x$

35. a. $\frac{33}{65}$ **b.** $\frac{16}{65}$ **c.** $-\frac{33}{56}$ **d.** $\frac{24}{25}$ **e.** $\frac{2\sqrt{13}}{13}$ **36. a.** $-\frac{63}{65}$ **b.** $-\frac{56}{65}$ **c.** $\frac{63}{16}$ **d.** $\frac{24}{25}$ **e.** $\frac{5\sqrt{26}}{26}$

37. a. 1 **b.** $-\frac{3}{5}$ **c.** undefined **d.** $-\frac{3}{5}$ **e.** $\frac{\sqrt{10+3\sqrt{10}}}{2\sqrt{5}}$ **38. a.** 1 **b.** $\frac{4\sqrt{2}}{9}$ **c.** undefined **d.** $\frac{4\sqrt{2}}{9}$ **e.** $-\frac{\sqrt{3}}{3}$

39. $\frac{\sqrt{3}}{2}$ **40.** $-\frac{\sqrt{3}}{3}$ **41.** $\frac{\sqrt{2-\sqrt{2}}}{2}$ **42.** $2-\sqrt{3}$ **43.** $\frac{1}{2}[\cos 2x - \cos 10x]$ **44.** $\frac{1}{2}[\sin 10x + \sin 4x]$ **45.** $-2\sin x \cos 3x$

46. $\frac{\sqrt{6}}{2}$ **47.** Proofs may vary. **48.** Proofs may vary. **49. a.** $y = \cot x$ **b.** Proofs may vary.

50. $x = \frac{2\pi}{3} + 2n\pi$ or $x = \frac{4\pi}{3} + 2n\pi$, where n is any integer. **51.** $x = \frac{\pi}{4} + 2n\pi$ or $x = \frac{3\pi}{4} + 2n\pi$, where n is any integer.

52. $x = \frac{7\pi}{6} + 2n\pi$ or $x = \frac{11\pi}{6} + 2n\pi$, where n is any integer. **53.** $x = \frac{\pi}{6} + n\pi$, where n is any integer. **54.** $\frac{\pi}{2}, \frac{3\pi}{2}$ **55.** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}$

56. $\frac{3\pi}{2}$ **57.** $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ **58.** π **59.** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **60.** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **61.** $0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$ **62.** $0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}$ **63.** $0, \pi$

64. 3.7890, 5.6358 **65.** 0.6847, 2.4569, 3.8263, 5.5985 **66.** $\frac{\pi}{4}$, 1.2490, $\frac{5\pi}{4}$, 4.3906 **67.** 0.8959, 2.2457

68. $t = \frac{2}{3} + 4n$ or $t = \frac{10}{3} + 4n$, where n is any integer. **69.** 12° or 78°